Evaluating array resolution

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***Abstract*—Both the size and shape of an array determine the resolution of which we can distinguish different signals and sources. By inspecting the beampattern of the array, we can analyse the resolution for different array configurations and different input frequencies. The resolution however is both a product of mainlobe width and side lobe level.**

***Index Terms*—Beampattern, array resolution, half power beamwidth, maximum side lobe level, acoustic eraser**

INTRODUCTION

THE resolution of a beamformer represents its capability to separate two incoming plane waves arriving at different angles accurately, thus assessing how well sources can be distinguished. The choice of array geometry and sensor weights all affect this capability. Often resolution is defined according to measuring the width of the array beampattern's mainlobe. Mainlobe width decides how accurately the array is able to determine a wave's direction of propagation, and this width is inversely related to array geometry and size. In addition to mainlobe width, also the maximum side lobe level should be taken into consideration when assessing the overall performance of an array. Whereas the overall size generally determines the width of the mainlobe, the number, position and weighting of microphones will generally determine side lobe level.

I. MAINLOBE WIDTH

A. Rayleigh criterion

One classical definition of resolution is the Rayleigh criterion, which states that two incoherent plane waves, propagating in two slightly different directions, are resolved if the mainlobe peak of the beampattern for one wave, falls on the first zero of the beampattern of the other. Or said in other words, the mainlobe width is given as the angular distance between the mainlobe peak and the first zero (or half the mainlobe width).

Looking at Fig. 1 we can see the beampattern of the array for two incoming plane waves of equal frequency and equal strength, but at different angles. The angles are chosen so that for that specific frequency, the mainlobe peak of the beampattern of the array from one wave, falls on the first zero of the beampattern of the array for the other. The resolution in this case is given as the angle between the two incoming waves, or in Fig. 1 approximately 10 degrees.

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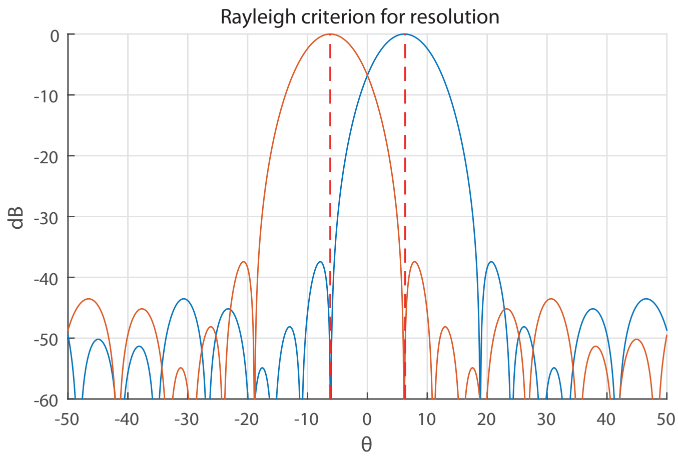


Fig. 1. Rayleigh criterion for resolution of two waves of equal strength and equal frequency. The mainlobe peak of the beampattern for one wave falls on the first zero of the beampattern of the other.

B. Half power beamwidth (HPBW)

The width of the mainlobe measured between the points where the beampattern power has been halfed, the half power beamwidth (HPBW), is another useful measure of mainlobe width. HPBW is the distance between the angles for which the array yields -3 dB attenuation, that is, the power from the peak of the beampattern has dropped 3 dB as seen in Fig 2. The distance between the angles defines the two sided opening angle of the array. The beampattern is dependent on array configuration, and the two sided opening angle will be frequency dependent, and can theoretically be used to determine the resolution of the array for different frequencies.

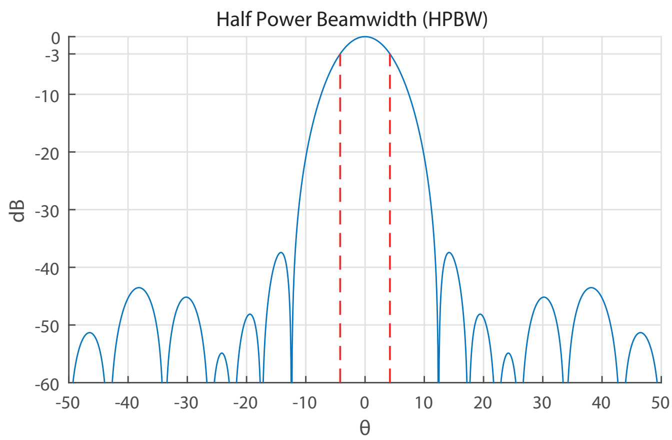


Fig. 2. Two-sided opening angle for half power beamwidth.

Rather than plotting the beampattern for a single frequency only, we can also look at the array response for angles

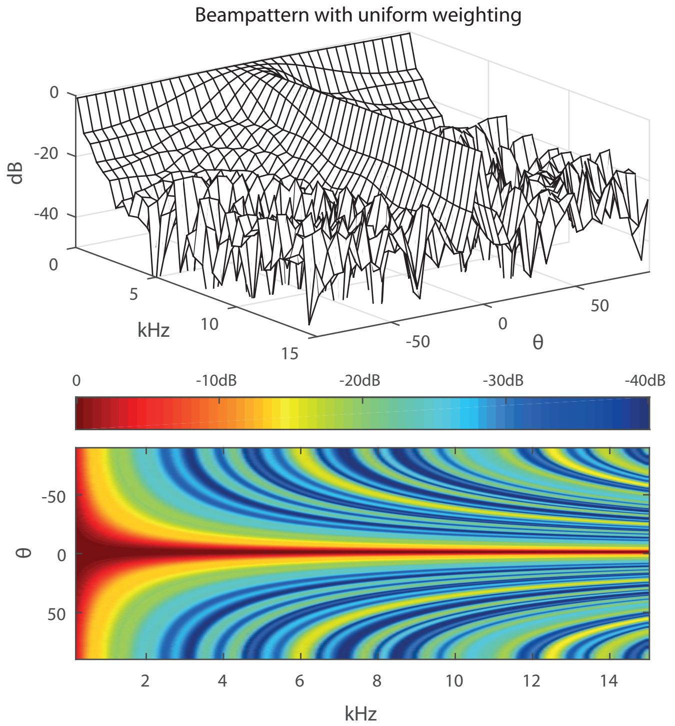


Fig. 3. Beampattern for all frequencies for Nor848A-10 with uniform weighting of elements

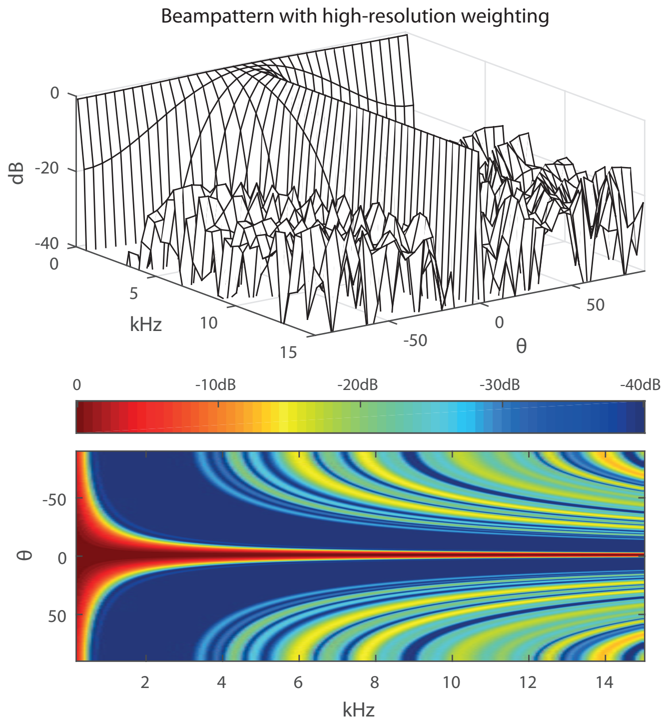


Fig. 4. Beampattern for all frequencies for Nor848A-10 with high-resolution weighting of elements

$\theta = \pm 90^\circ$ for a wide range of frequencies as seen in Fig. 3. Here the beampattern is plotted for frequencies between 200 Hz and 15 kHz for the Nor848A-10 with uniform weighting of elements. Both the top and the bottom figure convey the same information. A vertical slice from one of the figures at a certain frequency will then be the beampattern for that specific frequency (like in Fig. 2). As can be seen from Fig. 3, the mainlobe width gets wider for lower frequencies, especially below 1 kHz. A larger mainlobe width means that the two sided opening angle (HPBW) also will increase, and the resolution will then be worse for lower frequencies.

However to increase the resolution further, it is also possible to weight the individual microphones differently. That is we can apply a certain weight or gain that is different for different microphones. The goal is to get as narrow mainlobe as possible over a wide range of frequencies, and at the same time get the levels of the side lobes as low as possible. Seen in Fig. 4 is the same array and same frequencies as in Fig. 3, however we now apply high resolution weighting instead of uniform weighting of elements. It is easy to see how this has a dramatic effect on the beampattern, especially for the lower frequencies.

II. Resolution as a function of opening angle

By calculating the two sided opening angle we can plot the resolution as a function of frequency for different array geometries. Since resolution is inversely related to array size, an array that is larger in size will have better resolution than a smaller array. Seen in Fig. 5 is the two sided resolution at HPBW for the Nor848A-4 (40 cm, 128 microphones), Nor848A-10 (1.0 m, 256 microphones) and Nor848A-16 (1.6 m, 384 microphones). The HPBW resolution is plotted in linear scale at the top, and in logarithmic scale on the bottom. As seen from the figure there exist a linear dependence between resolution and frequency when plotted in logarithmic scale.

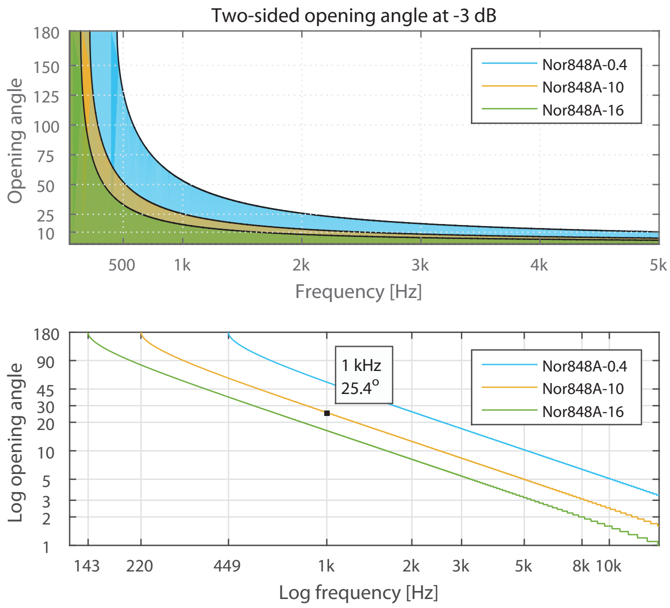


Fig. 5. Two-sided opening angle (HPBW) as a function of frequency for three different array sizes. The two-sided opening angle at 1 kHz is marked for the Nor848-10 at 25.4°. On the bottom part of the figure both opening angle and frequency is plotted in logarithmic scale.

For beamforming the resolution at the opening angle of an array is determined by the array size D and the frequency f, or wavelength λ of the received signal

$$HPBW \propto \frac{\lambda}{D} = \frac{c}{f \cdot D} \quad (1)$$

where c is the sound speed of the wave. Since there exist an almost linear relationship in logarithmic scale between size and opening angle, as seen in the bottom part of Fig. 5, it is possible to get the best theoretical straight line fit to the analytical graph by using for instance a least squares method. In that sense we can be able to get the theoretical best-fit line of the resolution from an equation rather than looking at a graph. The result of such an approach is shown in Fig 6.

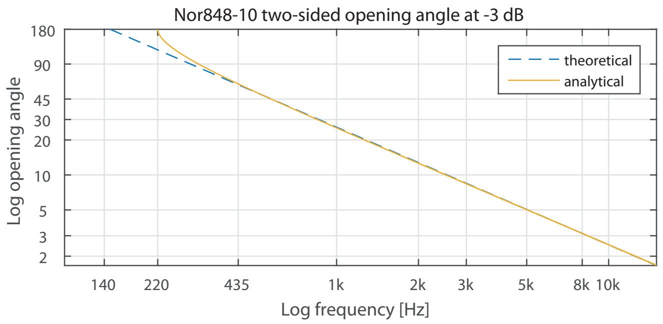


Fig. 6. The best linear fit by the least squares method to the opening angle as a function of frequency

As seen from the figure the correlation is very good for higher frequencies, but deviates as the frequency gets lower. Using the constants produced by the least square fit of the resolution, it is then possible to calculate the opening angle for any frequency by using the following equation

$$\begin{aligned} \log HPBW(f) &= \alpha + \beta * \log f \\ HPBW(f) &= 10^{\alpha + \beta * \log f} \\ HPBW(f) &= 10^{\alpha} \cdot f^{\beta} \end{aligned} \quad (2)$$

with the constants given in Table I as

TABLE I

	α	β	Valid for
Nor848A-0.4	4.7995	-1.0236	f > 780 Hz
Nor848A-10	4.4698	-1.0195	f > 400 Hz
Nor848A-16	4.2816	-1.0206	f > 260 Hz

where β is the slope of the line. The validity of using the equations is set so that the difference should not be greater than 1° between the analytical (simulated) resolution and the theoretical least squares fit. For instance by using the same data point as in Fig. 5 we have that the HPBW resolution of Nor848A-10 at 1 kHz is equal to

$$HPBW(1000 \text{ Hz}) = 10^{4.4698} \cdot 1000^{-1.0195} = 25.78^{\circ} \quad (3)$$

which is quite close to its analytical value of 25.4°.

III. Resolution of two incoherent sources

In Fig. 5 the two sided opening angle for the Nor848A-10 is marked at 1 kHz. By using this array, the measured HPBW resolution at 1 kHz is 25.4°. Remember that in this context what is meant by resolution is the minimum distance at which two incoherent sources of equal frequency and of equal intensity must be spaced from each other, while being at the same distance from the array, and still be resolved.

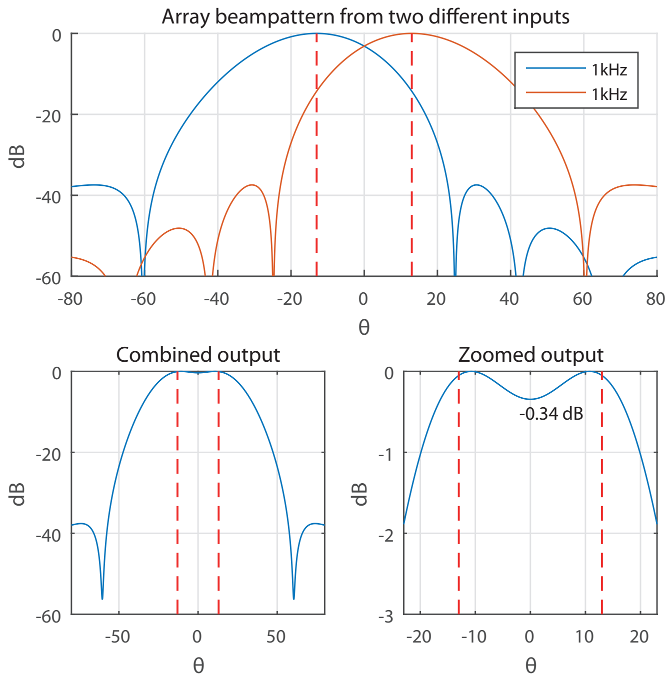


Fig. 7. Two equally strong incoherent sources at 1 kHz placed 25.4° apart. Bottom left shows the overall output of the array when the input consists of both sources, with bottom right displaying a zoomed in view.

Seen in Fig 7, two incoherent sources at 1 kHz are simulated hitting the Nor848-10 array with the incidence angle between the two sources at exactly 25.4° (incidence angle at $\pm 12.7°$). As seen in Fig 5 this was the minimum angle between sources at 1 kHz in order to distinguish them. On the top of the figure is the beampattern of the array with the two individual sources as input, with incidence angle marked in red. On the bottom part of the figure is the combined output of the array. The output equals the total beampattern of the array when the input is two incoherent sources at 1 kHz with their respective incidence angles as shown on the top of the figure. On the bottom right, the output is zoomed in to a dynamic range of only 3 dB. As can be seen from the figure the drop in the center of the two peaks of the output is -0.34 dB, which is enough to distinguish the individual sources.

IV. Resolution as a function of distance

By having the opening angle it is easy to extend this result to range resolution. The relationship between opening angle and range to give us the minimum distance between sources, is given as

$$R = \theta_f \cdot r \quad (4)$$

where R is the minimum distance between sources, θ_f is the two sided opening angle in radians at a given frequency f, and r is the distance from the array to the source. Or by using the results in (2) directly, the minimum distance between sources can also be calculated as

$$R = \frac{r \cdot 10^{\alpha} \cdot f^{\beta} \cdot \pi}{180} \tag{5}$$

where r is the measurement distance, f the measurement frequency, and the constants α and β are found in Table I.

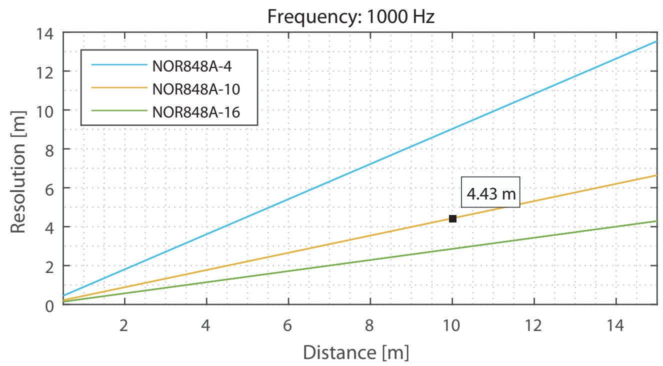


Fig. 8. Resolution as a function of distance for three different array sizes at 1 kHz

In Fig 8 the minimum distance that two equally strong incoherent sources must be spaced in order to be resolved, is plotted for distances between 0 to 15 m for an input frequency of 1 kHz for all three arrays. For lower frequencies the minimum distance will be larger, and for higher frequencies the minimum distance will be smaller. In essence, as the input frequency increases, two equally strong sources can be placed closer to one another and still be resolved. In the figure, the minimum distance between sources at 1 kHz at a range of 10 m when using the Nor848A-10 is marked. According to (4) this distance is calculated to be 4.43 m.

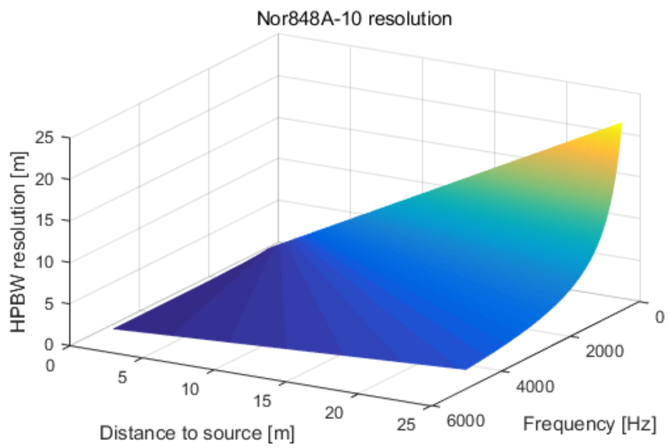


Fig. 9. Resolution as a function of distance and frequency

It is also possible to plot the resolution as a function of both distance and frequency as seen in Fig. 9. As the distance to the object of interest is near, the resolution is very good for all frequencies. However as the distance gets larger, and frequency gets lower, the minimum distance to distinguish sources increases as well. In essence the best results are achieved with a larger aperture, operating at a high frequency at a distance that is close to the source.

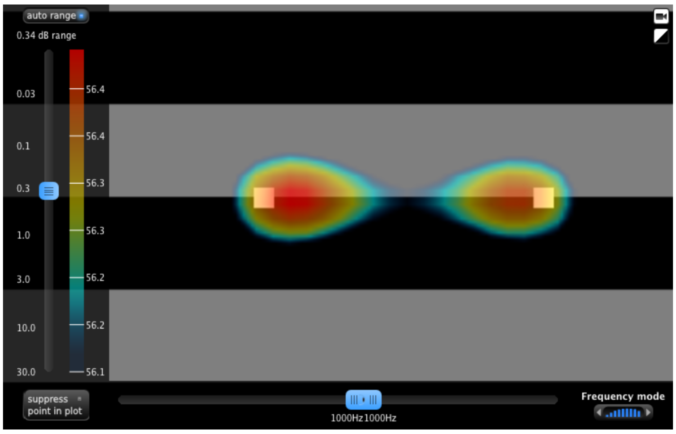


Fig. 10. Resolution of Acoustic Camera. Two incoherent sources at distance 10 m with 4.43 m between sources simulated with Nor848A-10 with uniform weighting of elements.

In Fig. 10 two incoherent sources at 1 kHz are simulated at a range of 10 m, with 4.43 m between sources using the Nor848A-10 array. The dynamic range of the plot which is adjusted by the colorbar on the left is set at 0.34 dB. By comparing Fig. 10 with Fig. 7 it is easy to see how the combined output on the lower part of Fig. 7 is shown in the acoustic camera software. The dynamic range set by the colorbar in the software goes down to 0.01. This in turn may suggest that even though the HPBW criterion states a certain minimum resolution angle, that angle could probably be even smaller and we would still be able to distinguish the two sources as long as the combined output has a drop of at least 0.01 dB.

V. Evaluating Array Performance with Real Input

A. Single frequency input

A natural extension based on the previous results would be to perform the same tests on an array with real input instead of simulated signals. This could then be used in order to compare the quality and correctness between different array configurations and different acoustic camera manufacturers at various frequencies. A common test case would then be to take two sound sources, for instance speakers or mobile phones, place them some distance apart at a certain range from the array, and let both speakers have the same single frequency output. The result from such a test method is shown in Fig 11 and Fig. 12.

As seen from the figures, quite dissapointing results, and not at all what we expected after seing the results given in Fig. 7 and Fig. 10. We have just stated that we can calculate the minimum opening angle, and get the minimum distance between two inchoerent sources of equal strength. However, when we in a real test environment place those sources in accordance with the theory, we are totally unable

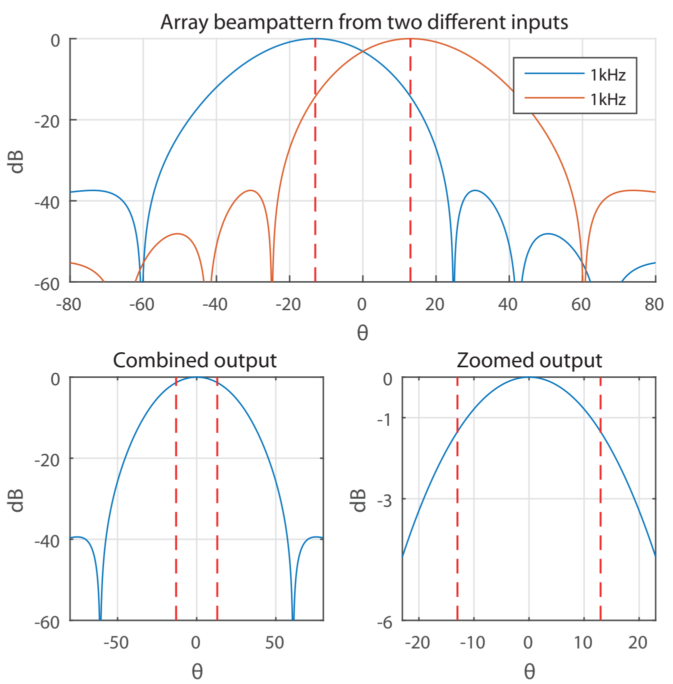


Fig. 11. Two equally strong coherent sources at 1 kHz placed 25.4° apart. Bottom left shows the overall received beampattern from both sources, with bottom right displaying a zoomed in view.

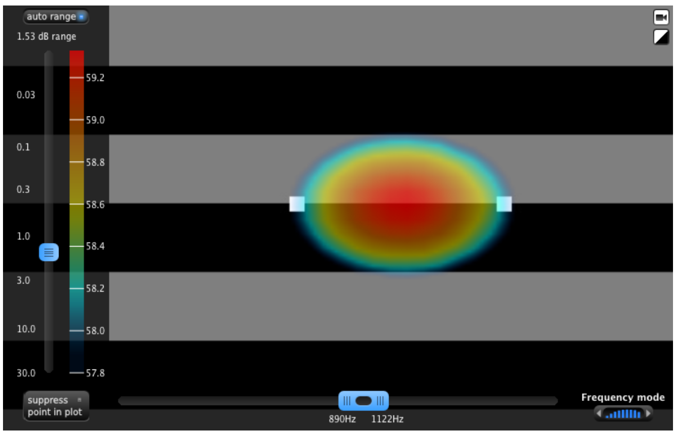


Fig. 12. Resolution of Acoustic Camera. Two coherent sources at distance 10 m with 4.68 m between sources.

to distinguish them with the array. Instead what is seen is a single source positioned in the middle of the two spakers. So what is going on here?

The key word here is that the resolution criterion is based on incoherent waves. Two wave sources are coherent if they have a constant phase difference and the same frequency. This is an ideal property of waves that enables stationary interference, which is basically an addition of wave functions. What is happening in a real life test environment is that two speakers or mobile phones with the same single frequency output, will be strongly coherent and will be perfectly superimposed on one another. The two sources will then generate a synthetic sound field that is hitting the array, just like an ultrasound array transducer is creating an imaging pulse by combining the output of several hundred small microphones. What is seen on the screen when we use the conventional delay-and-sum beamformer is exactly what is hitting the array, a complex sound field generated by two different sources with the same output.

Schröeder *et al.* [1] have argued that beamforming is mainly a broadband method, and that the widely adopted "narrowband view" of traditional acoustics does not apply to beamforming systems and best results are achieved in the broadband case. They concluded that *"It is shown that the widespread use of sinusoidal test signals which is appropriate in many other fields of acoustics will badly mislead array designers and end users if an objective evaluation of an individual beamforming system is the goal. The reason is that two monofrequent test signals of the same frequency are always perfectly linearly dependent (...), and this strong signal coherence thus will forcibly drive the simple delay and sum beamforming method to its very edge of failing instead of objectively evaluating its true performance potential. Therefore, a different approach is needed which should basically consider beamforming as a broadband method instead".*

B. Broadband input

A different approach to the single frequency sine input, is to use broadband noise sources and then filter on various frequencies in the software. By placing two loudspeakers some distance apart from one another at a certain measurement distance, and then use the acoustic camera and the acoustic camera software to analyse the results, we can get the pictures seen in Fig. 13 and Fig. 14, with the images obtained by the Nor848A-10 1.0m camera, and the Nor848A-0.4 40cm camera respectively. As seen from the figures the results are as expected, a larger aperture in size leads to higher resolution when incidence angle and input is the same.

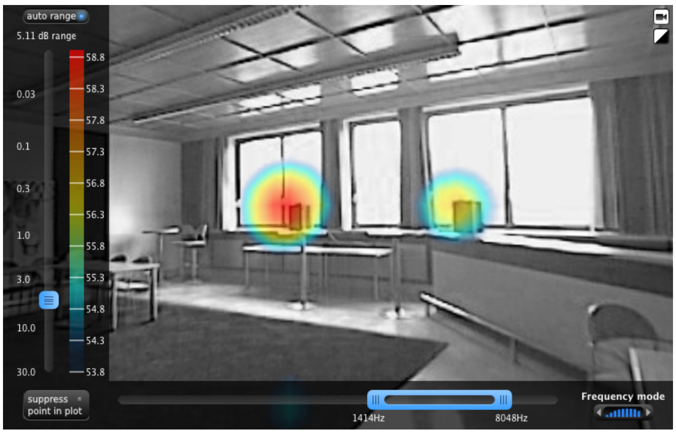


Fig. 13. Resolution of two loudspakers with white noise output by using Nor848A-10 1.0m camera

C. Using acoustic eraser to remove coherent sources

Seen in the Fig. 15, the same input signal as in Fig. 12 is being used. However this time the acoustic eraser feature of the Nor848A software is enabled and placed over one of the

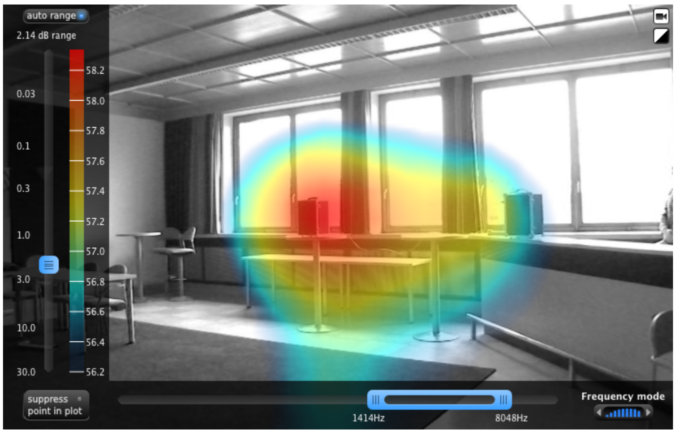


Fig. 14. Resolution of two loudspakers with white noise output by using Nor848A-0.4 40cm camera

sources. As seen from the figures, this enables us to correctly pin-point the position of even perfectly coherent sources.

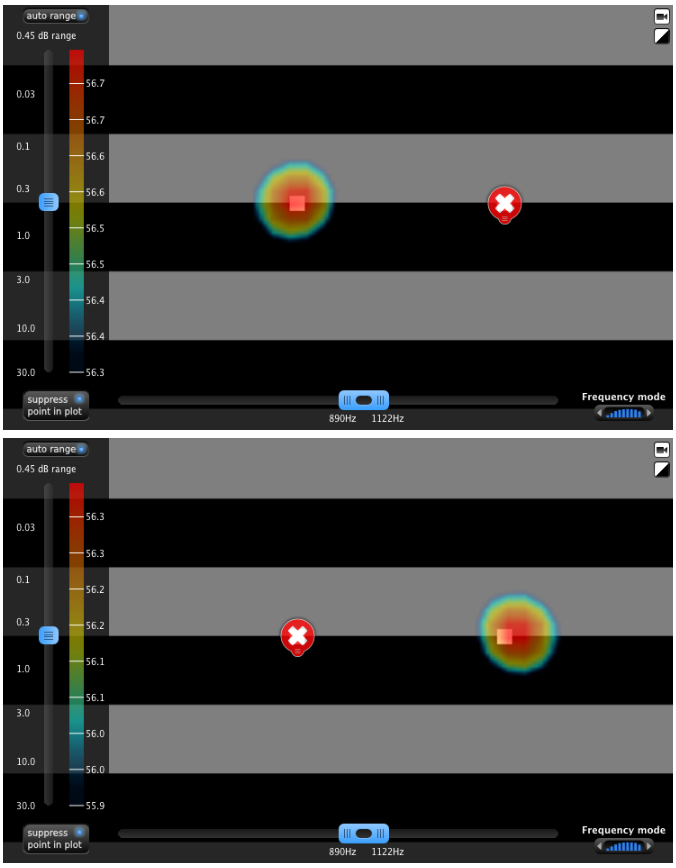


Fig. 15. Using the acoustic eraser to discover two perfectly coherent sources

Going back to the results seen in Fig. 14, in order to achieve even better resolution and being able to distinguish the sources with the Nor848A-0.4 as well, we could either try to filter the recording at a higher frequency band, or move the camera itself closer to the source. But as seen in Fig. 15 we also have the possibility to overcome the resolution limit by using the acoustic eraser. Seen in Fig. 16, this approach for live measurements is shown. Here the measurement set-up is exactly the same as that shown in Fig. 14, but in this case by using the acoustic eraser we are clearly able to pin point the individual sources.

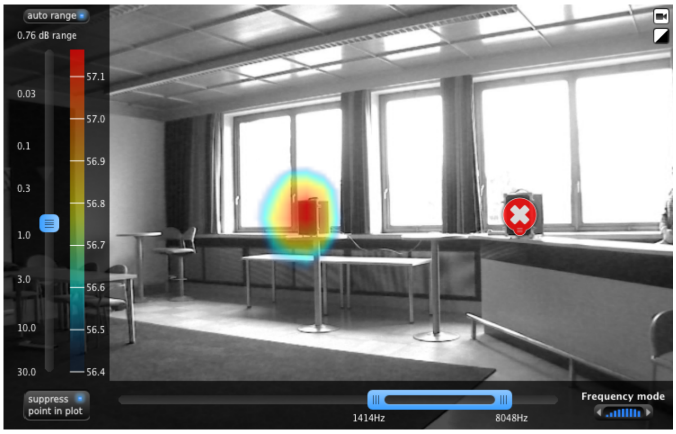


Fig. 16. Resolution of two loudspakers with white noise output by using Nor848A-0.4 40cm camera and acoustic eraser

VI. Maximum side lobe level (MSL)

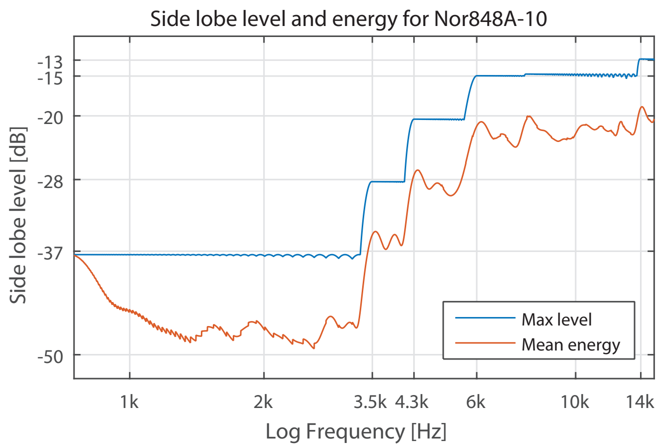


Fig. 17. Maximum side lobe level and mean side lobe energy as a function of frequency for the Nor848A-10 acoustic camera

Whereas the mainlobe width is mainly given by the overall size of the array, the maximum side lobe level (MSL) compared to the mainlobe is mainly decided by number of microphones, and the position and weighting of those microphones. Seen in Fig. 17 is the maximum side lobe level for the Nor848A-10 over all frequencies. Had the weighting of elements been equal, the MSL would have been around -13 dB regardless of frequency. Also the mean side lobe energy, that is the energy from other directions than the mainlobe, is also plotted. Clearly a large number of microphones has a huge impact on the low side lobe level. High side lobes will lead to waves arriving at directions other than that of the direction of the mainlobe to leak into the measurement. This will in turn produce so called ghost images or ghost spots - you measure a source that does not exist. To really see how the large side lobes influence the results, lets compare the Nor848A-10 with a ring array with same number of elements and same size. The ring array then has 256 microphones placed evenly in a circle with diameter of 1.0 m as seen in Fig 18.

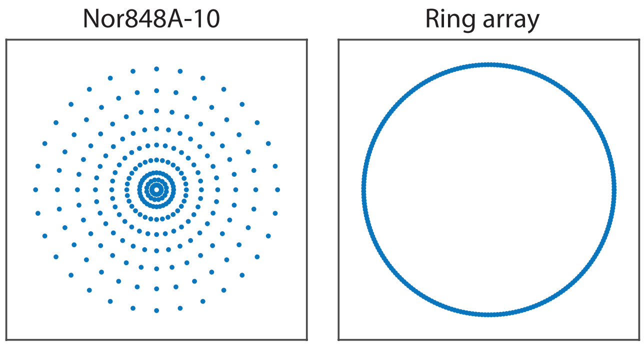


Fig. 18. Array geometry of Nor848A-10 and ring array with same number of elements and same diameter.

We can now calculate and analyze the beampattern at frequency $f = 3$ kHz for the two different arrays as seen in Fig. 19, with a zoomed version of the same beampattern seen in Fig. 20.

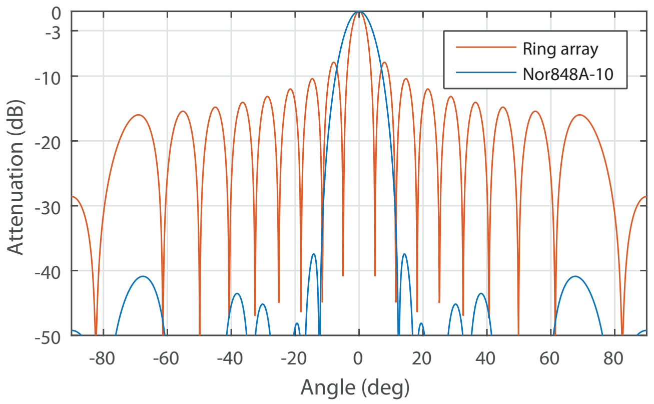


Fig. 19. Beampattern of Nor848A-10 and ring array with same number of elements and same diameter simulated at 3 kHz.

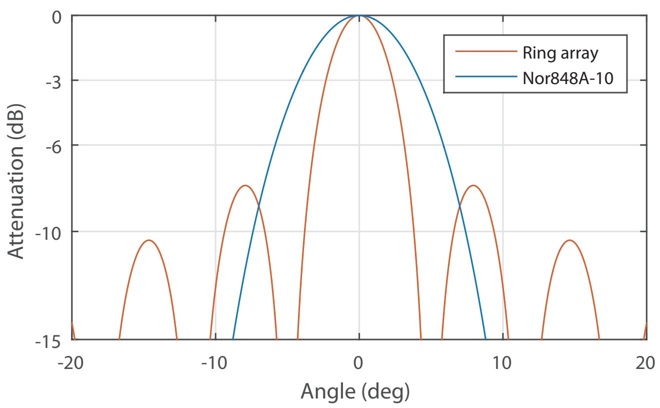


Fig. 20. Zoomed in view of beampattern at 3 kHz. The half power beamwidth of the ring array is more narrow than for the Nor848A-10.

Now judging from the beampattern alone, and according to the HPBW standard, the ring array should have the best resolution as the opening angle at -3 dB is more narrow than the same opening angle for the Nor848A-10. To see how this will influence an acoustic image, we can simulate the array output from the two different arrays for a custom test signal of our choice. Seen in Fig. 21 is just such an approach where the two different arrays are scanning over all incidence angles looking for nine point sources of equal strength and equal frequency $f = 3$ kHz. The dynamic range of the picture has been set equal for both images at 4 dB. As we can see from Fig. 21, both arrays are able to pin point the location of the different point sources, however the accuracy is better for the ring array at the same dynamic range. This is logical since the mainlobe of the beampattern at that frequency is more narrow for the ring array than for Nor848A-10.

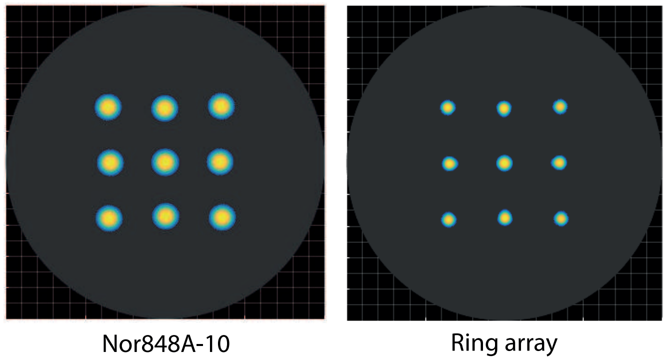


Fig. 21. Simulation of array output for Nor848A-10 and ring array with nine point sources with frequency 3 kHz and equal power as input. Dynamic range in the image is set to 4 dB.

Now however, let's change the dynamic range of the image to 8 dB instead of 4 dB. By doing this we get the images shown in Fig. 22

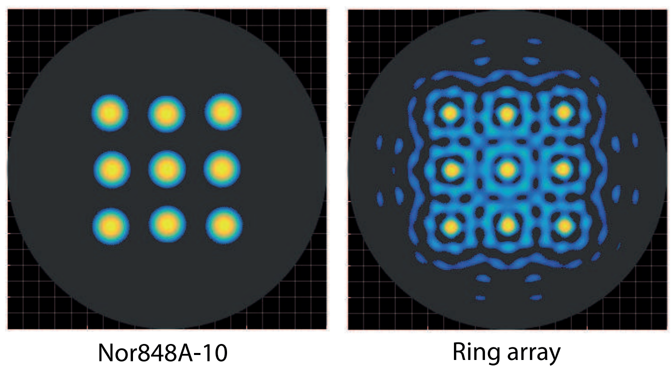
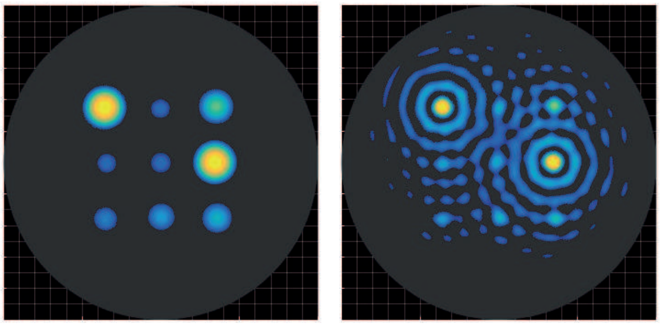


Fig. 22. Simulation of array output for Nor848A-10 and ring array with nine point sources with frequency 3 kHz and equal power as input. Dynamic range in the image is set to 8 dB.

Now we are seeing the effects of the high side-lobe levels much more clearly for the ring array than for Nor848A-10. The strength of the first side-lobe of the ring array was only around 8 dB lower than the mainlobe, which means that when displaying the image with dynamic range of 8 dB, power from side-lobes are leaking in to the image, and smearing the resolution. This is the so called ghost-spot effect.

But if dynamic range of 4 dB was sufficient to pin-point the sources accurately, why display it with 8 dB dynamic range? Remember that the images above were made when scanning for sources emitting soundwaves of equal strength. In real life situations this almost never happens, as different sources will have different sound power. So a more appropriate test would be to vary the strength of the individual point sources. Seen in Fig. 23 is the result from such an approach, where the same nine point sources are used, however now the strength differ by as much as 10 dB for different point sources. Clearly for the ring array, the strong sources leak in to the measurement and obscure the smaller sources, making it difficult to distinguish them.



Nor848A-10 Ring array

Fig. 23. Simulation of array output for Nor848A-10 and ring array with nine point sources with frequency 3 kHz and differences in power as input. Dynamic range in the image is set to 11 dB.

Although the ring array had better resolution according to the HPBW standard, the maximum side lobe level also has a huge impact on image quality. When evaluating array performance, low side lobe level in addition to narrow mainlobe is of utmost importance.

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